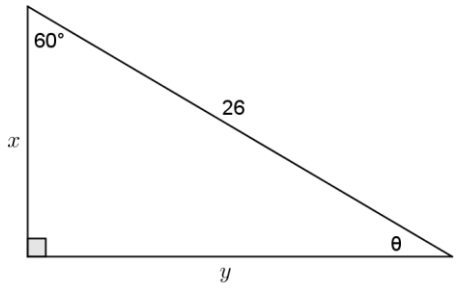
Youngstown State University
Trigonometry Final Exam Review
(Math 1511)

1. Convert each angle measure to decimal degree form. (Round your answers to thousandths place).
   a) $75°54'30"$
   b) $-145°18"$
2. Convert each angle measure to $D°M'S"$
   a) $3.5°$
   b) $-0.535°$
3. Rewrite each angle in degree measure. (Do not use a calculator.)
   a) $\frac{4\pi}{3}$
   b) $\frac{5\pi}{6}$
4. Find the length of the arc on a circle of radius r intercepted by a central angle θ.
   a) $r = 13\ inches,\ \theta = 240°$
   b) $r = 3\ meters,\ \theta = 1\ radian$
5. Find the area of the sector of a circle of radius r intercepted by a central angle θ. (Round answers to hundredths place.)
   a) $r = 15\ inches,\ \theta = \frac{\pi}{3}\ radians$
   b) $r = 1.5\ feet,\ \theta = 225°$
6. Find the distance between the cities***: Dallas, Texas with latitude*** $\mathbf{32°47'9"N}$ ***and Omaha, Nebraska with latitude*** $\mathbf{41°15'50"N.}$ Assume that Earth is a sphere of radius 4000 miles and that the cities are on the same longitude (one city is due north of the other). Round your answer to the nearest mile.
7. Find the exact values of $x, y\ and\ \theta$.



8. You are standing 45 meters from the base of a building. You estimate that the angle of elevation to the top of the 86th floor (the observatory) is 82°. (Round your answers to tenths places).
   a) If the total height of the building is another 125 meters above the 86th floor, what is the approximate height of the building?
   b) One of your friends is on the 86th floor. What is the distance between you and your friend?
9. The point $(-7, -2)$ is on the terminal side of an angle θ in standard position. Determine the exact values of the six trigonometric functions of the angle θ.

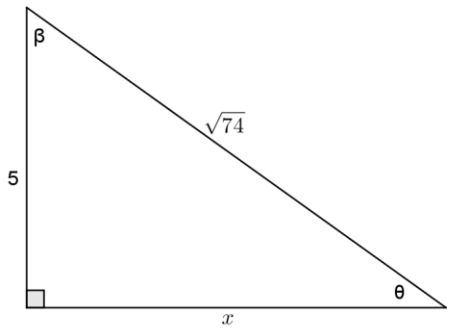
10. Find the values of the six trigonometric functions of θ with the given $cos\theta = \frac{4}{5}$ and $tan\theta < 0$.

11. Evaluate sine, cosine, and tangent of the following measurements of the angle β without a calculator.
    a) $\beta = 300°$
    b) $\beta = \frac{5\pi}{6}$

12. Find the point (x, y) on the unit circle that corresponds to the real number t.
    a) $t = \frac{7\pi}{6}$
    b) $t = \frac{8\pi}{3}$
    c) $t = \frac{3\pi}{2}$
13. Sketch the graph of the function. (Include two full periods.) Label asymptotes where appropriate.
    a) $y = \frac{1}{5}\cos x$
    b) $y = -sin\frac{2\pi x}{3}$
    c) $y = 2sin(x - \frac{\pi}{2})$
    d) $y = -4\cos(6x + \pi)$
    e) $y = 3\cos\left(x + \frac{\pi}{4}\right) + 3$
    f) $y = -2tan5x$
    g) $y = \frac{1}{2}secx$
    h) $y = csc\frac{x}{4}$
    i) $y = 2\cot(x + \frac{\pi}{2})$
14. Evaluate the expression without using a calculator.
    a) $arc\cos\left(\frac{\sqrt{3}}{2}\right)$
    b) $\sin^{-1}\left(\frac{-\sqrt{2}}{2}\right)$
    c) $arc\tan\left(-\sqrt{3}\right)$
15. Find the exact value of the expression.
    a) $\sec(arc\ \sin\left(\frac{4}{5}\right))$
    b) $\cos(\tan^{-1} 2)$

16. For the right triangles below find the measurements of the missing side length, x and angles, $\theta\ and\ \beta$.

a)



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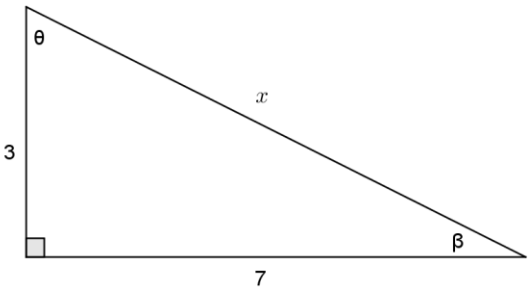


17. A ship leaves port at noon and has a bearing of $N\ 40°\ E$. The ship sails at 15 knots. How many nautical miles north and how many nautical miles east will the ship have traveled by 4:00 p.m.? (Round to nearest tenths)

18. A plane 90 miles south and 110 miles east of London Heathrow airport. What bearing should be taken to fly directly to the airport?

19. The sun is 25° above the horizon. Find the length of a shadow cast by a building that is 100 feet tall. Round your answer to hundredths place.

20. A passenger in an airplane at an altitude of 10 kilometers sees two towns directly to the east of the plane. The angles of depression to the towns are 21° and 56°. How far apart are the towns? Round your answer to hundredths place.

21. An engineer erects a 95-foot cellular telephone tower. Find the angle of elevation to the top of the tower at a point on level ground 70 feet from its base. Round your answer to hundredths place.

22. A cellular telephone tower that is 300 feet tall is placed on top of a mountain that is 1300 feet above sea level. What is the angle of depression from the top of the tower to a cell phone user who is 5 horizontal miles away and 300 feet above sea level? Round your answer to hundredths place.

23. The length of a shadow of a tree is 170 feet when the angle of elevation of the sun is 17°. Approximate the height of the tree. Round your answer to hundredths place.

24. Given $\cos\left(\frac{\pi}{2} - x\right) = \frac{20}{29}$ and $\cos x = \frac{21}{29}$, find the values (if possible) of all six trigonometric functions. (If an answer is undefined, write UNDEFINED.

25. Given $\csc\theta = -4$ and $\cos\theta < 0$, find the values (if possible) of all six trigonometric functions. (If an answer is undefined, write UNDEFINED.

26. Use the fundamental identities to simplify the expressions:
    a) $4\tan(-x)\cos x$
    b) $9\sin\phi\,(\csc\phi - \sin\phi)$
    c) $5\cos\left(\frac{\pi}{2} - x\right)\sec x$

27. Use the trigonometric substitution $x = 5\cos\theta$ to write the algebraic expression $\sqrt{25 - x^2}$ as a trigonometric function of θ, where $0 < \theta < \frac{\pi}{2}$.

28. Verify the following identities:
    a) $\tan t \cot t = 1$
    b) $4\cos x + 4\sin x \tan x = 4\sec x$
    c) $7\cos^2\beta - 7\sin^2\beta = 14\cos^2\beta - 7$
    d) $\frac{\tan^2\alpha}{\sin\alpha} = \sec\alpha\tan\alpha$
    e) $\frac{5}{\tan\theta} + 5\tan\theta = \frac{5\sec^2\theta}{\tan\theta}$
    f) $\frac{1+\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = 2\sec x$
    g) $\frac{9}{\cos\beta+1} + \frac{9}{\cos\beta-1} = -18\csc\beta\cot\beta$
    h) $\tan\left(\frac{\pi}{2} - x\right)\tan x = 1$
    i) $(3 + 3\sin y)[3 + 3\sin(-y)] = 9\cos^2 y$

29. Solve (in radians) the following equations and use n as an integer constant:
    a) $\tan x + \sqrt{3} = 0$
    b) $5\cos x + 3 = -\cos x$
    c) $64\cos^2 x - 16 = 0$
    d) $10\sin^2 2x = 5$
    e) $2\cos 2x - 1 = 0$
    f) $2\cos\frac{x}{3} - \sqrt{2} = 0$
    g) $9\sec^2 x - 9 = 0$

30. Find all solutions (if possible) of the following equations in the interval $[0,2\pi)$. If there is no solution, write NO SOLUTION.
    a) $10\sin^2 x = 10 + 5\cos x$
    b) $6\sin x + \csc x = 0$
    c) $16\sin^2 x + 24\sin x + 8 = 0$

31. Find the exact value of the expressions:
    a) $\sin\frac{\pi}{20}\cos\frac{\pi}{5} + \cos\frac{\pi}{20}\sin\frac{\pi}{5}$
    b) $\cos\frac{5\pi}{12}$

32. Find the exact value of the trigonometric expressions $\sin(u+v)$ given that $\sin u = \frac{7}{25}$ and $\cos v = -\frac{3}{5}$. (Both u and v are in Quadrant II).

33. Find the exact solutions of the following equations in the interval $[0,2\pi)$.
    a) $5\sin 2x \sin x = 5\cos x$
    b) $\cos 2x + \sin x = 0$

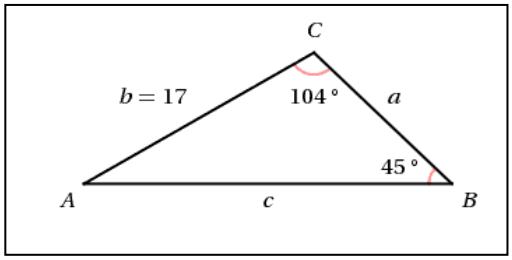
34. Find the exact values of $\sin 2u$, $\cos 2u$, $\sin\frac{u}{2}$, $\cos\frac{u}{2}$ for each, given: $\sin u = -\frac{4}{5}$, $\frac{3\pi}{2} < u < 2\pi$

35. Use the power-reducing formulas as many times as possible to rewrite the expression $7\cos^4 x$ in terms of the first power of the cosine.

36. Use the half-angle formula to simplify $\sqrt{\frac{1-\cos(10x)}{2}}$.

37. Use the Law of Sines to solve the triangle. Round your answers to hundredths place

    a)



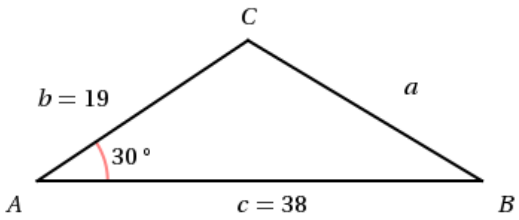
    b) $A = 38°, B = 74°, c = 29.1$

    c) $B = 12°30', a = 13, b = 10$

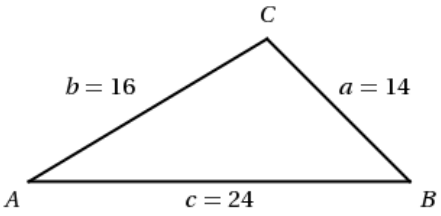
38. Use the Law of Sines to solve (if possible) the triangle. If two solutions exist, find both. Round your answers to hundredths places.
    a) $A = 117°, a = 120, b = 93$
    b) $A = 78°, a = 13, b = 20$
    c) $A = 51°, a = 10.9, b = 12.5$

39. Use the Law of Cosines to solve the triangle. Round your answers to hundredths place.

    a)

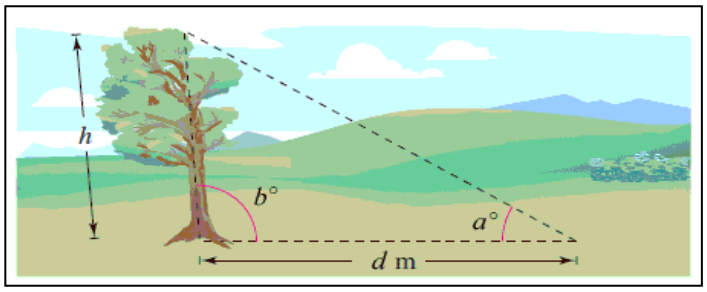


    b)



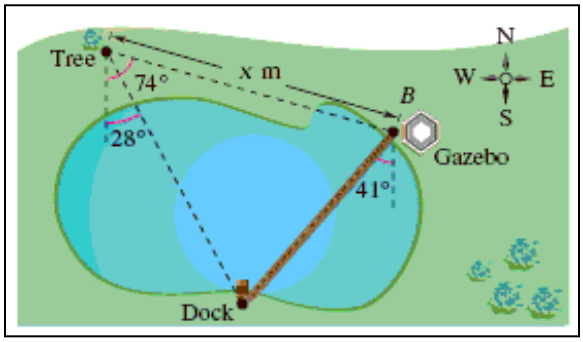
    c) $A = 100°, b = 9, c = 10$

40. Because of prevailing winds, a tree grew so that it was leaning 1° from the vertical. At a point $d = 42$ meters from the tree, the angle of elevation to the top of the trees is $a = 32°$ (see figure). Find the height h of the tree. (Round your answer to tenths place.)

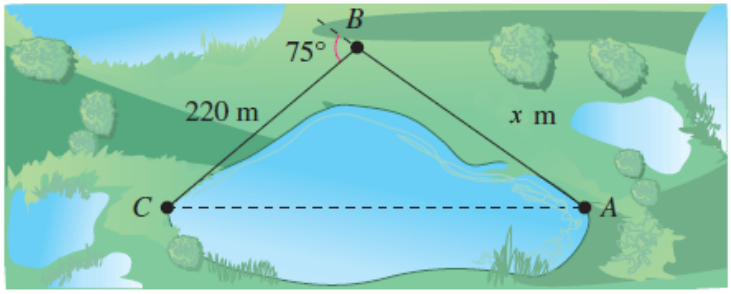


41. A boat race runs along a triangular course marked by buoys $A, B, and\ C$. The race starts with the boats headed west for 4000 meters. The other two sides of the course lie to the north of the first side, and their lengths are 1600 meters and 2900 meters. Draw a figure that gives a visual representation of the situation, and find the bearings for the last two legs of the race. (Round your answer to tenths place.)

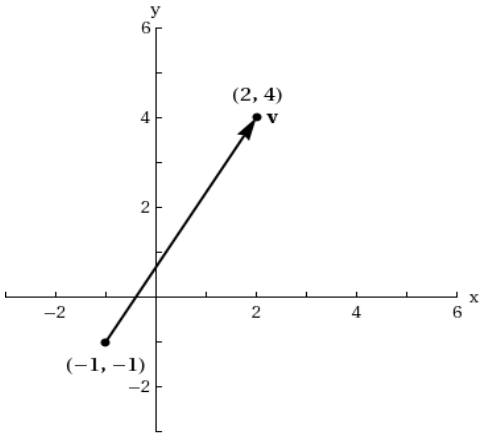
42. A bridge is to be built across a small lake from a gazebo to the dock (see figure). The bearing from the gazebo to the dock is $S\ 41^\circ\ W$. From a tree $x = 110$ meters from the gazebo, the bearings to the gazebo and the dock are $S\ 74^\circ\ E$ and $S\ 28^\circ\ E$, respectively. Find the distance from the gazebo to the dock. (Round your answer to the nearest whole number.)



43. To approximate the length of a marsh, a surveyor walks $x = 420$ meters from point A to point B, then turns 75° and walks 220 meters to point C (see figure). Approximate the length AC of the marsh. (Round your answer to tenths place.)



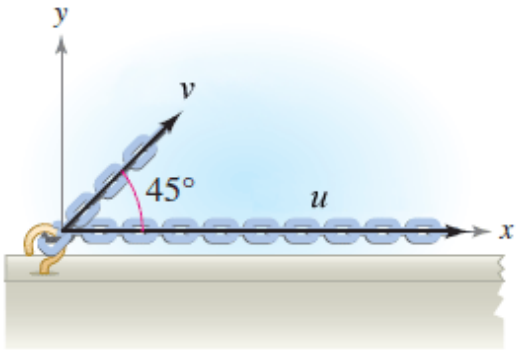
44. Find the component form and the magnitude of the vector $\boldsymbol{v}$.



45. Find the vector $\boldsymbol{v}$, given the magnitude $\|v\| = 75$ and the same direction as $\boldsymbol{u} = \langle -7, 24 \rangle$.

46. Find the component form (exact values) of $\boldsymbol{v}$ given its magnitude and the angle it makes with the positive x-axis.
    a) $\|v\| = 2,\ \theta = 45^\circ$
    b) $\|v\| = \frac{9}{8},\ \theta = 150^\circ$

47. Forces with magnitudes of $\boldsymbol{v} = 160$ newtons and $u = 305$ newtons act on a hook (see figure). The angle between the two forces is 45°. Find the direction and magnitude of the resultant of these forces. (round your answers to two decimal places.)



48. Represent the complex number graphically. Then write the trigonometric form of the number with $0 \leq \theta \leq 2\pi$. Use exact values.
    a) $4 + 4i$
    b) $3 - \sqrt{3}i$
    c) $-2i$
    d) $-7+6\text{i}$
49. Plot the points in polar coordinates.
    a) $\left(4, \frac{5\pi}{4}\right)$
    b) $\left(-3, \frac{2\pi}{3}\right)$
50. Convert the polar coordinate $\left(-3, \frac{5\pi}{6}\right)$ to a rectangular coordinate (x, y).
51. Convert the rectangular coordinate $(-3, -3)$ to a polar coordinate $(r, \theta), r > 0, 0 \leq \theta \leq 2\pi$.