MATHEMATICS 571 **Answers to Final Examination Review Problems**

1. a) $2a^2 + 4ab + 2b^2 - 5a - 5b$ b) $4x^2$

2. a) $x \geq 4$ or $x \leq -1$ b) $x \neq 0, 1, -1$

3. 3 units to the left and 4 units down

4. a) $f \circ g(x) = \dfrac{1}{x-1},\ x \geq 0, x \neq 1;$ $g \circ f(x) = \dfrac{1}{\sqrt{x^2-1}},\ x > 1,$ or $x < -1$

   b) $f \circ g(x) = \dfrac{x^2}{(x-1)^2} + 1,\ x \neq 1;$ $g \circ f(x) = \dfrac{x^2+1}{x^2},\ x \neq 0$

5. a) $(0,0)$, no symmetry; b) no intercepts; origin

   c) $(2,0),\ (-2,0),\ (0,2)$; y-axis

6. a) DNE b) 3

7. a) -1 b) $-\dfrac{1}{2}$ c) 4 d) $-\infty$

   e) $+\infty$ f) DNE g) 1 h) -1

   i) DNE j) 0 k) 1/3 l) DNE

   m) 0 n) 1 o) 1/3 p) 4

   q) $\sqrt{3}/2$ r) $-\sin x$

8. a) 15 b) 9/2

9. a) $(-\infty, \infty)$ b) $(-\infty, -3]$ and $(-3, \infty)$

10. a) $x = 1,\ y = 1$ b) $x = -2,\ y = 1$ c) $y = 1,\ y = -1,\ x = 0$

    d) $x = 2,\ y = 1$

11. $f(x) = \dfrac{x^2-1}{x-1},\ a = 1.$ $f(1)$ DNE, but $\lim\limits_{x \to 1} f(x) = 2$

12. -112 13. $\dfrac{4}{9}$

14. $\dfrac{73}{24}$ 15. $\dfrac{-8}{3}$

16. $y = 3x$ 17. $(2, 6)$

18. $y + 3 = (22/21)(x - 2)$

19. $-\sqrt{3}/9$

20. $3\sqrt{2}/8$

21. $-\sqrt[3]{4}/12$

22. $1/2$

23. $-8/3$

24. $3\sqrt{3}/4$

25. $(4 + 3\sqrt{2})/(3 + 2\sqrt{2}) = \sqrt{2}$

26. $-\sqrt{3}$

27. $-9/2$

28. $[3/4, \infty)$

29. $(-3/2, 1]$

30. $(-4, 0),\ (3, \infty)$

31. $(-\infty, -2)$ *and* $(0, 2)$

32. $3,\ -4$

33. None

34. At $x = 2\sqrt{2} - 3$

35. At $x = 0$

36. a) 0 and -16

36. b) 16 and 0

37. 192 ft, 6 sec, -128 ft/sec

38. 64 ft/sec, 288 ft

39. 5000 ft^2

40. 80

41. 5 ft on each side

42. Overland for 5/8 mile

43. 2 in, 128 in^3

44. $33\sqrt{5}$ mph

45. 1 ft/sec

46. 39 ft/min

47. 0.0702, 0.07

48. $\pm\, 0.06$ ft^3

49. $7 + 1/14$

50. $0.04M$

51. 15 meters

52. a) Yes, the function $f(x) = x^5 + 2x^2 - 10x + 5$ is continuous on $[1, 2]$ and $f(1) < 0 < f(2)$.

b) Yes, the function$f(x) = x - \sqrt{x}$ is continuous on $[4, 9]$ and $f(4) < 5 < f(9)$.

c) Yes, the function $f(x) = x - \cos x$ is continuous on $[0, \pi]$ and $f(0) < 0 < f(\pi)$.

d) No, the function $f(x) = x - \tan x$ is not continuous at $\dfrac{\pi}{2}$.

53. $f(x) = \dfrac{2}{3}x^3 + \dfrac{3}{2}x^2 - 8x + \dfrac{47}{6}$

54. $f(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 + 2x$

55. a) 4

b) 3

c) $f(t)$ is increasing in $[1, 4]$ and decreasing in $[0, 1]$ and $[4, 5]$

d) $(2, 2)$

56. f', the derivative of the function shown, is increasing on $[0, 2]$ and is decreasing in $[2, 5]$, as can be seen by noting the concavity of the function, which indicates the sign of the derivative of $f'(t)$.

57. a) When time is 12 seconds.

b) 16 meters

c) 4 meters

d) 12 meters

e) Forward; foot on the gas

f) Backward; foot on the brake

58.

a) $\frac{1}{30}(3x^2 + 4)^5 + C$

b) $\frac{1}{16}(2x^4 + 4x)^4 + C$

c) $\frac{195}{64}$

d) $-\frac{3}{x} - \frac{3}{x^2} - \frac{2}{3x^3} + C$

e) $\frac{38}{3}$

f) $\frac{1}{3}(3x^2 - 2x + 1)^{3/2} + C$

g) $\dfrac{-1}{3(x+5)^3}+C$

h) $\dfrac{-1}{12(10x^2+2x+40)^3}+C$

i) $\dfrac{1}{9}(6x^3+1)^{3/2}+C$

j) $\dfrac{2}{5}(2-x)^{5/2}-\dfrac{4}{3}(2-x)^{3/2}+C$

k) $-\dfrac{3}{4}(3+z^{-1})^{4/3}+C$

l) $\sin(\sec x)+C$

m) $-\dfrac{3}{4}\left(\cos\dfrac{x}{3}\right)^4+C$

n) $\dfrac{1}{2}(\tan x)^2+C$

59. a) x^2-8 b) $2x(x^4+6)^{5/2}$

60. $\dfrac{4}{15}$

61. $\dfrac{9}{2}$

62. $\dfrac{48}{5}$

63. 64π

64. $\dfrac{96\pi}{5}$

65. a) 8π b) $\dfrac{256\pi}{15}$

66. $\dfrac{100\pi}{3}$

67. $\dfrac{250}{3}$

68. $\dfrac{\pi}{240}$

69. $\dfrac{5}{2}$

70. $\dfrac{2}{\pi}$

71. a) 28125π b) 21093.75π

72. a) 840 b) 952.5 c) 862.5

73. $1L, 2K, 3B, 4A, 5C, 6I, 7D, 8E, 9G, 10P, 11J, 12M, 13O, 14Q, 15R, 16H$