1. For the function f defined by $f(x) = 2x^2 - 5x$ find the following:

   a) $f(a+b)$ b) $f(2x) - 2f(x)$

2. Find the domain of g if

   a) $g(x) = \sqrt{x^2 - 3x - 4}$ b) $g(x) = \dfrac{x+2}{x^3 - x}$

3. The graph of $f(x) = \sqrt{x+3} - 4$ is the same as the graph of $f(x) = \sqrt{x}$ except that it is moved how?

4. For the functions f and g, find $f \circ g$, $g \circ f$, and the domains of each.

   a) $f(x) = \dfrac{1}{x^2 - 1}, \quad g(x) = \sqrt{x}$ b) $f(x) = x^2 + 1, \quad g(x) = \dfrac{x}{x-1}$

5. Find the intercepts of the following equations. Also determine whether the equations are symmetric with respect to the y-axis or the origin.

   a) $y = x^4 + x^3 + x^2$ b) $y = \dfrac{1}{x^3 - 3x}$

   c) $y = 2 - |x|$

6. Determine

   a) $\lim_{x \to 1} f(x), \text{ if } f(x) = \begin{cases} 4x - 2, & \text{if } x < 1 \\ 7, & \text{if } x = 1 \\ 2x - x^2, & \text{if } x > 1 \end{cases}$

   b) $\lim_{x \to -2} g(x), \text{ if } g(x) = \begin{cases} x + 5, & \text{if } x \geq -2 \\ 2x^2 - x - 7, & \text{if } x < -2 \end{cases}$

7. Determine

   a) $\lim_{x \to 2} (\sqrt{x-1} - \sqrt{3x-2})$ b) $\lim_{x \to -1} \dfrac{x^3 + 1}{x^2 - 4x - 5}$

   c) $\lim_{x \to \infty} \dfrac{4x^3 + x - 5}{x^3 - 2}$ d) $\lim_{x \to 7^-} \dfrac{x^2 + 49}{x - 7}$

e) $\displaystyle\lim_{x\to 7^+} \frac{x^2+49}{x-7}$

f) $\displaystyle\lim_{x\to 7} \frac{x^2+49}{x-7}$

g) $\displaystyle\lim_{x\to \frac{1}{2}^+} \frac{|2x-1|}{2x-1}$

h) $\displaystyle\lim_{x\to \frac{1}{2}^-} \frac{|2x-1|}{2x-1}$

i) $\displaystyle\lim_{x\to \frac{1}{2}} \frac{|2x-1|}{2x-1}$

j) $\displaystyle\lim_{x\to -\infty} \frac{x^2+4x-1}{x^4}$

k) $\displaystyle\lim_{x\to -\infty} \frac{\sqrt[3]{x^3+2x^2+1}}{3x-5}$

l) $\displaystyle\lim_{x\to -3^+} \sqrt{x^2-9}$

m) $\displaystyle\lim_{x\to \infty} \frac{\sin(\frac{1}{x})}{x}$

n) $\displaystyle\lim_{x\to \frac{\pi}{2}^-} \frac{\sec x - 1}{\tan x}$

o) $\displaystyle\lim_{x\to 1} \frac{\sqrt{8+x^2}-3}{x-1}$

p) $\displaystyle\lim_{h\to 0} \frac{\tan(\frac{\pi}{3}+h)-\tan\frac{\pi}{3}}{h}$

q) $\displaystyle\lim_{h\to 0} \frac{\sin(\frac{\pi}{6}+h)-\sin(\frac{\pi}{6})}{h}$

r) $\displaystyle\lim_{h\to 0} \frac{\cos(x+h)-\cos x}{h}$

8. Define P such that the following functions are continuous at $x = 3$.

$$f(x) = \begin{cases} \dfrac{3(x^4-81)}{x^2-9}, & \text{if } x \neq 3 \\ Px+9, & \text{if } x = 3 \end{cases} \qquad g(x) = \begin{cases} \dfrac{x^3-27}{x^2-9}, & \text{if } x \neq 3 \\ P, & \text{if } x = 3 \end{cases}$$

9. Determine the intervals on which the functions defined below are continuous.

a) $f(x) = \begin{cases} 8-7x, & \text{if } x \leq 4 \\ -x-16, & \text{if } x > 4 \end{cases}$

b) $f(x) = \begin{cases} x^2+1, & \text{if } x \leq -3 \\ 5-x, & \text{if } x > -3 \end{cases}$

10. Identify all asymptotes of the following.

a) $y = \dfrac{x-2}{x-1}$

b) $y = \dfrac{x^2-3x+2}{x^2-4}$

c) $y = \dfrac{\sqrt{x^2+4}}{x}$

d) $y = \dfrac{x^2-9}{x^2-5x+6}$

11. Give a specific example to show that it is possible for $\displaystyle\lim_{x\to a} f(x)$ to exist if $f(a)$ is undefined.

12. Determine y' when $x = 2$ if $y = \dfrac{7}{\sqrt{x^4-15}}$.

13. Determine $f'(5)$ if $f(x) = \sqrt[3]{(2x+17)^2}$.

14. Determine y' when $x = 1$ if $y = \dfrac{8x}{3} - \dfrac{3}{8x}$.

15. Determine y' when $x = 1$ if $y = (2 - x)\sqrt{x^2 + 8}$.

16. Find an equation of the tangent line to the curve defined by $y = 2x^2 - 5x + 8$ when $x = 2$.

17. At what point (x, y) is the tangent line to the curve $y = 2x^2 - 5x + 8$ parallel to the line $y = 3x - 7$

18. Find the slope of the line tangent to the graph of $x^2 + 2xy^2 + 3y = 31$ at the point $(2, -3)$.

19. Let $y = \sqrt{x^2 - 1}$. Find y'' when $x = 2$.

20. If $f(x) = \sqrt{x + \sqrt{x}}$, find $f'(1)$.

21. Determine $f'(1)$ if $f(x) = \sqrt[3]{\dfrac{x}{x^3 + 1}}$.

22. Determine $f'(3)$ if $f(x) = \dfrac{1}{x - \sqrt{x^2 - 5}}$.

23. Determine y'' at $x = 1$ if $y = 3\sqrt[3]{x^4} - \dfrac{1}{3x^3}$.

24. Let $y = [\cos(2x - \pi)]^3$. Find y' at $x = \frac{\pi}{6}$.

25. Determine $f'(\frac{\pi}{4})$ if $f(x) = \dfrac{\tan x}{1 + \cos x}$.

26. If $f(x) = \sin 3x \cos 2x$, find $f'(\frac{\pi}{6})$.

27. Let $y^4 + x^4 - 2x^2y + 9x = 9$. Find y' at $(1, 1)$.

28. Determine the intervals on which f is increasing if $f'(x) = x^2(4x - 3)$ and $f''(x) = 6x(2x - 1)$.

29. Let $f'(x) = \dfrac{x - 1}{2x + 3}$ and $f''(x) = \dfrac{5}{(2x + 3)^2}$. Determine the intervals on which f is decreasing.

30. Determine the intervals on which f is concave upward if

$$f(x) = \frac{1}{10}x^5 + \frac{1}{6}x^4 - 4x^3 + 87x + 69.$$

31. Let $f'(x) = \dfrac{-3}{2(x^2 - 4)^2}$ and $f''(x) = \dfrac{6x}{(x^2 - 4)^3}$. Determine the intervals on which f is concave downward.

32. Determine all points of inflection for $f(x) = \frac{1}{12}x^4 + \frac{1}{6}x^3 - 6x^2$.

33. Determine all points of inflection for $f(x) = \frac{1}{12}x^4 - \frac{7}{3}x^3 + \frac{49}{2}x^2 + 88$.

34. Let $f(x) = \frac{1}{3}x^3 + 3x^2 + x - 2$. Determine all local minima for f.

35. If $f(x) = \frac{1}{4}x^4 - \frac{5}{2}x^2 + 3$, find all local maxima for f.

36. Find the maximum and minimum values of the function $f(x) = x^3 - 12x$ on the interval $[0, 3]$. Repeat the problem for the interval $[-3, 0]$.

37. A rock thrown from the top of a cliff is $s(t) = 192 + 64t - 16t^2$ feet above the ground t seconds after being thrown. Determine the height of the cliff, the time it takes the rock to reach the ground, and the velocity of the rock when it strikes the ground.

38. A rock is thrown vertically upward from the roof of a house 32 feet high with an initial velocity of 128 ft/sec. What is the speed of the rock at the end of 2 seconds? What is the maximum height the rock will reach?

39. What is the maximum area which can be enclosed by 200 ft. of fencing if the enclosure is in the shape of a rectangle and one side of the rectangle requires no fencing?

40. A woman throws a ball vertically upward from the ground. The equation of its motion is given by $s(t) = -16t^2 + ct$, where c is the initial velocity of the ball. If she wants the ball to reach a maximum height of 100 ft., find c.

41. A rectangular open tank is to have a square base, and its volume is to be 125 yd^3. The cost per square yard for the base is \$8 and for the sides is \$4. Find the dimensions of the tank in order to minimize the cost of the material.

42. A power station is on one side of a river which is 1/2 mile wide, and a factory is 1 mile downstream on the other side of the river. It costs \$300 per foot to run power lines overland and \$500 per foot to run them under water. Find the most economical way to run the power lines from the power station to the factory.

43. A cardboard box manufacturer wishes to make open boxes from pieces of cardboard 12 in. square by cutting equal squares from the four corners and turning up the sides. Find

the length of the side of the square to be cut out in order to obtain a box of the largest possible volume. What is the largest possible volume?

44. A train leaves a station traveling north at the rate of 60 mph. One hour later, a second train leaves the same station traveling east at the rate of 45 mph. Find the rate at which the trains are separating 2 hours after the second train leaves the station.

45. A street light hangs 24 ft. above the sidewalk. A man 6 ft. tall walks away from the light at the rate of 3 ft/sec. At what rate is the length of his shadow increasing?

46. A barge is pulled toward a dock by means of a taut cable. If the barge is 20 ft. below the level of the dock, and the cable is pulled in at the rate of 36 ft/min, find the speed of the barge when the cable is 52 ft. long.

47. Find the values of Δy and dy if $y = 2x^2 - x$, $x = 2$, and $dx = \Delta x = 0.01$.

48. Use differentials to approximate the maximum possible error that can be produced when calculating the volume of a cube if the length of an edge is known to be 2 ± 0.005 ft.

49. Use differentials to approximate $\sqrt{50}$.

50. The moment of inertia of an annular cylinder is $I = .5M(R_2^2 - R_1^2)$, where M is the mass of the cylinder, R_2 is its outer radius, and R_1 is its inner radius. If $R_1 = 2$, and R_2 changes from 4 to 4.01, use differentials to estimate the resulting change in the moment of inertia.

51. The range of a shell shot from a certain ship is $R = 300\sin(2\theta)$ meters, where θ is the angle above horizontal of the gun when it is shot. If the gun is intended to be fired at an angle of $\pi/6$ radians to hit its target, but due to waves it actually shot .05 radians too low, use differentials to estimate how far short of its target the shell will fall.

52. In each of the following, determine whether the Intermediate Value Theorem guarantees the equation has a solution in the specified interval.

    a) $x^5 + 2x^2 - 10x + 5 = 0,\ [1, 2]$

    b) $x - \sqrt{x} = 5,\ [4, 9]$

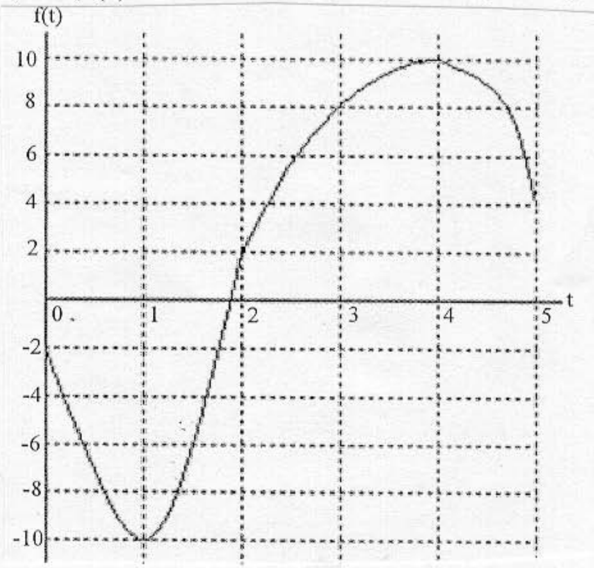
    c) $x = \cos x,\ [0, \pi]$

    d) $x = \tan x,\ [\pi/4, 3\pi/4]$

53. Find $f(x)$ if $f''(x) = 4x + 3, f(1) = 2$, and $f'(1) = -3$.

54. Find $f(x)$ if $f'(x) = x^2 + 3x + 2$, and $f(-3) = -\frac{3}{2}$.

For problems 55 and 56, let $f(t)$ be the function defined by the graph shown.



55. Estimate the following, getting numbers to the nearest integer.

    a. The instantaneous rate of change of f at $t = 3$.

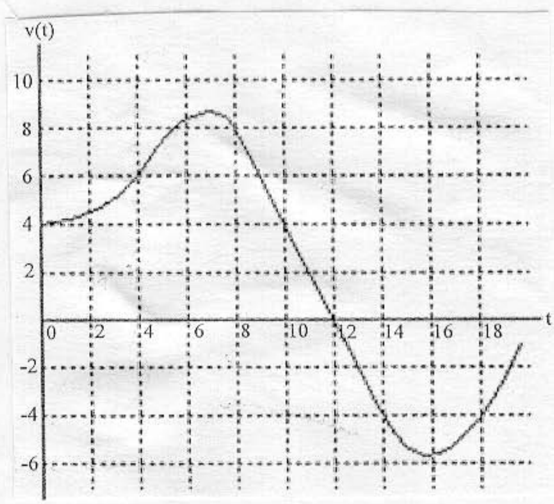
    b. The average rate of change of f over the interval $[0, 4]$.

    c. The intervals where $f(t)$ is increasing and where it is decreasing.

    d. The inflection point or points of $f(t)$.

56. Estimate the intervals where $f'(t)$ is increasing and where it is decreasing, getting numbers to the nearest integer.

57. Let $v(t)$, as shown in the graph, be the velocity of a car in meters per second at time t in seconds, where positive velocity means the car is moving forward. Estimate numbers to the nearest integer.



    a. When did the car stop?

    b. Approximately how far did the car travel in the time interval 8 to 12 seconds?

    c. Approximately how far did it travel in the time interval 12 to 14 seconds?

    d. Approximately how far did it travel in the time interval 8 to 14 seconds?

e. At the time 2 seconds, is the car moving forward or backward? Is the driver's foot on the gas or the brake?

f. At the time 16 seconds, is the car moving forward or backward? Is the driver's foot on the gas or the brake?

58. Integrate the following:

a) $\int x(3x^2+4)^4\ dx$

b) $\int (2x^4+4x)^3(2x^3+1)\ dx$

c) $\int_1^2 \frac{(2x^3+3x+5)}{x^5}\ dx$

d) $\int \frac{(3x^2+6x+2)}{x^4}\ dx$

e) $\int_3^8 \sqrt{12-x}\ dx$

f) $\int (3x-1)\sqrt{3x^2-2x+1}\ dx$

g) $\int \frac{1}{(x+5)^4}\ dx$

h) $\int \frac{(5x+\frac{1}{2})}{(10x^2+2x+40)^4}\ dx$

i) $\int 3x^2\sqrt{6x^3+1}\ dx$

j) $\int x\sqrt{2-x}\ dx$

k) $\int \frac{\sqrt[3]{3+z^{-1}}}{z^2}\ dz$

l) $\int \sec x \tan x \cos(\sec x)\ dx$

m) $\int \sin\frac{x}{3}\left(\cos\frac{x}{3}\right)^3\ dx$

n) $\int \tan x\,(\sec x)^2\ dx$

59. Differentiate the following:

a) $f(x) = \int_3^x t^2 - 8\ dt$

b) $f(x) = \int_4^{x^2} (t^2+6)^{5/2}\ dt$

60. Find the area of the region bounded by $y = x^2$ and $y = x^4$.

61. Find the area of the region bounded by $y = x^2$ and $y = 2 - x$.

62. Find the area of the region bounded by $y = x^4$ and $y = 8x$.

63. Find the volume of the solid generated by revolving the region bounded by $y = x^{3/2}$, the x-axis, and the line $x = 4$ about the x-axis.

64. Find the volume of the solid generated by revolving the region bounded by $y^2 = 4x$ and $x^2 = 4y$ about the x-axis.

65. Find the volume of the solid generated by revolving the region in the first quadrant bounded by $y = x^2$, $y = 4$, and the y-axis about a) the y-axis; b) $y = 4$.

66. Find the volume of the solid generated by revolving the region bounded by $y = x + \dfrac{4}{x}$, the x-axis, and the lines $x = 1$ and $x = 3$ about the y-axis.

67. A solid has for its base the region in the first quadrant bounded by $x^2 + y^2 = 25$. Every plane section of the solid taken perpendicular to the x-axis is a square. Find the volume of the solid.

68. A solid has as its base the region in the xy-plane bounded by the graphs of $y = x$ and $y^2 = x$. Find the volume of the solid if every cross section by a plane perpendicular to the x-axis is a semicircle with diameter in the xy-plane.

69. Find the average value of the function $f(x) = x^2 + x + 1$ on the interval $[-1, 2]$.

70. Find the average value of the function $f(x) = \sin x$ on the interval $[0, \pi]$.

71. A cylindrical water tank with a circular base has radius 3 feet and height 10 feet. How much work is required to empty the tank by pumping the water out of the top if a) the tank is full? b) the tank is half full? (Assume that the density of water is 62.5 lb/ft^3.)

72. A bucket with 24 lb of water is raised 30 feet from the bottom of a well. Find the work done assuming that a) the weight of the empty bucket is 4 lb and the weight of the rope is negligible; b) the bucket weighs 4 lb and the rope weighs 4 oz/ft; c) in addition, the water leaks out of the bucket at a constant rate and that only 18 lb are left at the top.

73. Match each numbered item with a lettered item. (There are more lettered items than numbered items. Some lettered items don't match any numbered item.)

1. Definition of $\lim_{x\to a} f(x) = L$.
2. Definition of $\lim_{x\to a^+} f(x) = L$.
3. Definition of $\lim_{x\to\infty} f(x) = L$.
4. Definition of "f is continuous at a".
5. The Intermediate Value Theorem.
6. Definition of the derivative of f at a.
7. Definition of a differentiable function f at a.
8. Theorem relating differentiability and continuity.
9. The power rule for differentiation.
10. Definition of the differential.
11. Definition of a function f having an absolute maximum at c.
12. Definition of a function f having a local maximum at c.
13. The Extreme Value Theorem.
14. Definition of a function that is increasing on an interval I.
15. The Mean Value Theorem.
16. Definition of an antiderivative of f on an interval I.

A. $\lim_{x\to a} f(x) = f(a)$.

B. For every $\epsilon > 0$ there is a corresponding number N such that $|f(x) - L| < \epsilon$ whenever $x > N$.

C. If f is continuous on the closed interval $[a, b]$, and N is a number strictly between $f(a)$ and $f(b)$, then there exists a number c in (a, b) such that $f(c) = N$.

D. The limit $\lim_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$ exists.

E. If f is differentiable at a, then f is continuous at a.

F. If f is continuous at a, then f is differentiable at a.

G. $\dfrac{d}{dx}x^n = nx^{n-1}$.

H. The function g has the property $g'(x) = f(x)$ for all x in I.

I. $\lim_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$

J. $f(c) \geq f(x)$ for all x in the domain of f.

K. For every $\epsilon > 0$ there is a corresponding number $\delta > 0$ such that $|f(x) - L| < \epsilon$ whenever $a < x < a + \delta$.

L. For every $\epsilon > 0$ there is a corresponding number $\delta > 0$ such that $|f(x) - L| < \epsilon$ whenever $0 < |x - a| < \delta$.

M. There is an open interval I containing c such that $f(c) \geq f(x)$ for all x in I.

N. $f'(x) > 0$ for all x in I.

O. If f is continuous on $[a, b]$ then there are numbers c and d in $[a, b]$ such that $f(c)$ is an absolute maximum for f in $[a, b]$ and $f(d)$ is an absolute minimum for f in $[a, b]$.

P. If f is differentiable, $dy = f'(x)dx$.

Q. $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I.

R. If f is continuous on $[a, b]$ and differentiable in (a, b), then there is a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$