Math 1510

Youngstown State University
College Algebra Final Exam Review
(Math 1510)

1. Find all *Real solutions* for the following:
   a) $-x^2 + 5x = 6$
   b) $\frac{1}{9}x^2 - x - 18 = 0$
   c) $(x-12)^2 = 16$
   d) $4x = 8 - x^2$
   e) $x^2 + 4x = -5$
   f) $36x^3 = 64x$
   g) $1 + \frac{5}{x} = \frac{8}{x^2}$
   h) $3x^{1/3} + 4x^{2/3} = 7$
   i) $\sqrt{9-x} - 5 = 0$
   j) $(x+3)^{2/3} = 49$
   k) $\frac{8}{x+2} - \frac{6}{x+4} = 1$

2. Find all *Complex solutions* for the following:
   a) $\frac{3}{2}x^2 - 6x + 9 = 0$
   b) $x^4 + 7x^2 - 144 = 0$
   c) $x^3 + 2x^2 + 7x = -14$

3. Solve the following inequalities and express your answer in interval notation.
   a) $x^2 > 2x + 8$
   b) $x^3 - 4x \geq 0$
   c) $x^2 - 2x + 15 \leq 0$
   d) $\frac{5x-9}{x-9} \geq 0$

4. Write an equation of the lines through the given point that is parallel to and perpendicular to the given line.
   a) $2x + 3y = 5, \ \left(-\frac{1}{2}, \frac{5}{3}\right)$
   b) $x - 9 = 0, \ (5, -7)$

5. A pharmaceutical salesperson receives a monthly salary of \$4700 plus a commission of 3% of sales. Write a linear equation for the salesperson's monthly wage W in terms of monthly sales S.

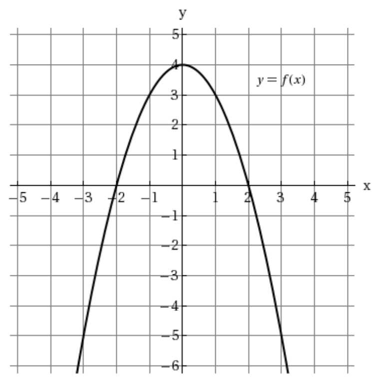
6. Given the function $g(t) = 5t^2 - 9t + 3$, evaluate the following and simplify:
   a) $g(2)$
   b) $g(t-2)$
   c) $g(t) - g(2)$

7. Given the function $f(x) = \begin{cases} 1-3x, & x \leq -2 \\ 0, & -2 < x < 2 \\ x^2+2, & x \geq 2 \end{cases}$ evaluate the following and simplify:
   a) $f(-3)$
   b) $f(4)$
   c) $f(-1)$

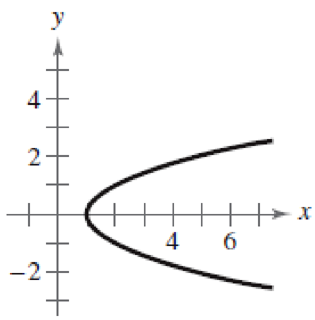
8. Find the domain for the following functions:
   a) $f(x) = 4x^2 + 2x - 5$
   b) $g(x) = \sqrt{x-14}$
   c) $f(s) = \frac{\sqrt{s-5}}{s-9}$

9. Use the graph of the function to find the domain and range of f. Then use the graph to find the following:

a) $f(-3)$
b) $f(2)$
c) $f(0)$
d) $f(3)$

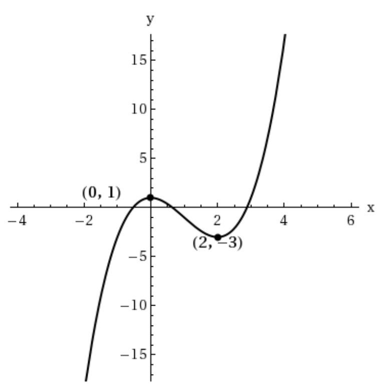


10. Determine whether the graph represents y as a function of x.



11. Determine the intervals on which the functions are increasing, decreasing, or constant.

a)



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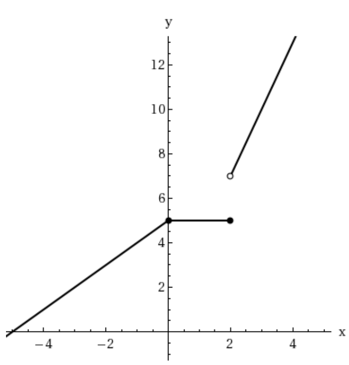


12. Sketch the graphs of the functions:

a) $g(x) = \begin{cases} x+1, & x \leq -5 \\ \frac{1}{2}x - 3, & x > -5 \end{cases}$

b) $f(x) = \begin{cases} 1-(x-4)^2, & x \leq 5 \\ \sqrt{x-5}, & x > 5 \end{cases}$

13. Sketch the graphs of the following functions showing the transformation from the key points of the parent function.

a) $g(x) = 3-(x+8)^2$
b) $f(x) = (x-1)^3 + 5$
c) $h(x) = -|x| - 5$
d) $f(x) = 4 - [\![x]\!]$
e) $g(x) = \sqrt{6-x} - 2$

14. Given $f(x) = x^2$ and $g(x) = 3x - 5$, find the following:

a) $(f+g)(x)$
b) $(f-g)(x)$
c) $(fg)(x)$
d) $(f/g)(x)$
e) Find the domain of f/g

15. Given $f(x) = \frac{6}{x}$ and $g(x) = \frac{6}{x^2}$, find the following:

a) $(f+g)(x)$
b) $(f-g)(x)$
c) $(fg)(x)$
d) $(f/g)(x)$
e) Find the domain of f/g

16. Given $f(x) = x^2$ and $g(x) = x - 5$, find the following:

a) $(f \circ g)(x)$
b) $(g \circ f)(x)$
c) $(g \circ g)(x)$

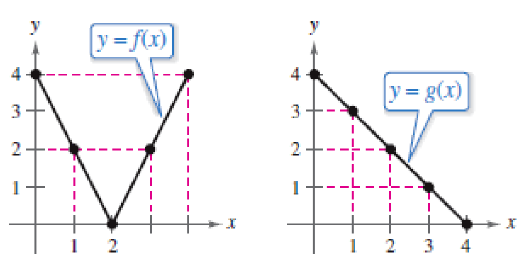
17. Given $(f \circ g)(x) = (2x-5)^2 + 4$ and $g(x) = 2x - 5$, find $f(x)$:

18. Given $f(x) = \sqrt{x+7}$ and $g(x) = x^2$, find the following:

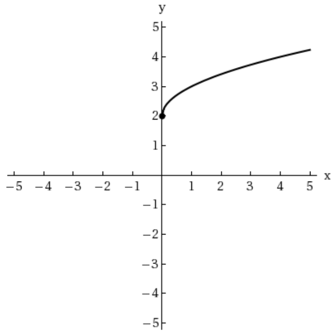
a) $(f \circ g)(x)$
b) $(g \circ f)(x)$
c) Domain of $f \circ g$
d) Domain of $g \circ f$

19. Use the graph of f and g to evaluate the functions:

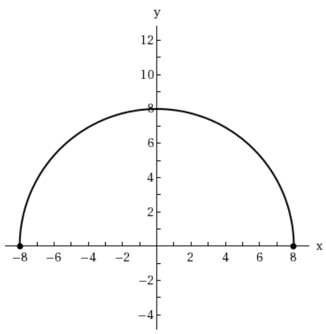
a) $(f+g)(1)$
b) $\left(\frac{f}{g}\right)(2)$
c) $(f \circ g)(2)$
d) $(g \circ f)(2)$



20. Use the graph of the function to sketch the graph of its inverse function $y = f^{-1}(x)$



21. Does the function defined by the graph have an inverse?



22. Determine whether the following functions have an inverse function. If it does, then find the inverse function (define domain of inverse function if necessary) .
    a) $f(x) = 8x + 9$
    b) $f(x) = (x + 1)^2, \ \ x \geq -1$

23. Write the function $f(x) = -x^2 - 4x + 1$ in standard form $y = a(x - h)^2 + k$ and determine the following special features of the graph.
    a) Vertex
    b) Axis of symmetry
    c) x-intercept(s)
    d) y-intercept

24. Write the function $f(x) = 8x^2 - 8x + 21$ in standard form $y = a(x - h)^2 + k$ and determine the following special features of the graph.
    a) Vertex
    b) Axis of symmetry
    c) x-intercept(s)
    d) y-intercept

25. Write the standard form $y = a(x - h)^2 + k$ of the equation of the parabola that has the indicated vertex and passes through the given point:
    a) Vertex $(6, -1)$; point $(4,7)$
    b) Vertex $\left(-\frac{1}{5}, \frac{5}{4}\right)$; point $(-1,0)$

26. Sketch the graph of the following polynomials using the Leading Coefficient Test, finding the real zeros and plotting sufficient solution points and drawing a continuous curve through the points:
    a) $g(x) = x^4 - 9x^2$
    b) $f(x) = -4x^3 + 16x^2 + 9x$
    c) $g(x) = -\frac{1}{4}(x - 2)^2(x + 2)^2$

27. Use long division to divide the following:
    a) $(10x^3 - 28x^2 + 41x - 20) \div (5x - 4)$
    b) $(x^3 - 27) \div (x - 3)$
    c) $(7x + 3) \div (x + 2)$

28. Find the rational zeros of the functions:
    a) $h(x) = x^3 - 12x^2 + 41x - 30$
    b) $f(x) = 9x^4 - 9x^3 - 58x^2 + 4x + 24$

29. Find all real solutions of the polynomial equations:
    a) $z^4 + z^3 + z^2 + 3z - 6 = 0$
    b) $x^4 - 73x^2 - 72x = 0$

30. Given the function $f(x) = \frac{1}{x-6}$ find the following:
    i. The domain
    ii. Intercepts (as points)
    iii. Vertical and horizontal asymptotes (as equations)
    iv. Plot additional solution points as needed to sketch the graph of f.

31. Given the function $f(x) = \frac{1-3x}{1-x}$ find the following:
    i. The domain
    ii. Intercepts (as points)
    iii. Vertical and horizontal asymptotes (as equations)
    iv. Plot additional solution points as needed to sketch the graph of f.

32. Given the function $f(x) = \frac{x}{x^2-25}$ find the following:
    i. The domain
    ii. Intercepts (as points)
    iii. Vertical and horizontal asymptotes (as equations)
    iv. Plot additional solution points as needed to sketch the graph of f.

33. Given the function $f(x) = \frac{3(x+4)}{x^2+x-12}$ find the following:
    i. The domain
    ii. Intercepts (as points)
    iii. Vertical and horizontal asymptotes (as equations)
    iv. Plot additional solution points as needed to sketch the graph of f.

34. Given the function $f(x) = \frac{x^2-16}{x}$ find the following:
    i. The domain
    ii. Intercepts (as points)
    iii. Vertical and slant asymptotes (as equations)
    iv. Plot additional solution points as needed to sketch the graph of f.

35. Write the equations of the circle in standard form $(x-h)^2 + (y-k)^2 = r^2$. Find the center and radius.
    a) $x^2 + y^2 - 2x + 12y + 36 = 0$
    b) $2x^2 + 2y^2 - 2x - 2y - 161 = 0$

36. Sketch the graph of the functions showing the horizontal asymptote:
    a) $g(x) = 2^x + 7$
    b) $g(x) = 5^{-x+2}$

37. Write the logarithmic equation $\log_4 16 = 2$ in exponential form.

38. Write the exponential equation $2^3 = 8$ in logarithmic form.

39. Evaluate the following:
    a) $\log_{64} 4$
    b) $\log_8 1$
    c) $\log_4 \frac{1}{16}$
    d) $\log_6 2 + \log_6 18$

40. Find the domain, x-intercept, and vertical asymptote of the functions. Then sketch the graphs.
    a) $h(x) = \log_4(x - 4)$
    b) $h(x) = \ln(x + 3)$

41. Write the following as a sum or difference of logarithms:
    a) $\log_2 \frac{1}{z^8}$
    b) $\ln \frac{\sqrt{x}y^3}{z^4}$

42. Write the following as a single logarithm:
    a) $\log_5 6 - \log_5 t$
    b) $2\ln 8 + 9\ln(z - 4)$

43. Solve the exponential equations. Give the exact form and then estimate to thousandths place (where appropriate)
    a) $3^{x+1} = 27$
    b) $5e^x = 71$
    c) $7^{-8t} = 0.90$
    d) $900e^{-5x} = 95$
    e) $\ln x - 4 = 0$

44. Find the number of years it takes $1000 to double if it is invested at an interest rate of 1.4% compounded continuously. (Round to hundredths place)

45. Find the interest rate on an initial investment of $300 that grew to $1005 after 10 years. (Round percent to hundredths place)

46. A radioactive isotope has a half-life (years) of 1599. Find the amount of the isotope after 1000 years with an initial quantity of $11g$.

47. Solve the system by the method of graphing $\begin{cases} x - 3y = -7 \\ x + 2y = 3 \end{cases}$

48. Solve the following systems by method of substitution. Check your solution graphically.

    a) $\begin{cases} 2x + y = 4 \\ x^3 - 4 + y = 0 \end{cases}$

    b) $\begin{cases} x^2 - y = 0 \\ x^2 + 4x + y = 0 \end{cases}$

49. Solve the following systems by method of elimination.

a) $\begin{cases} 5x + 3y = 6 \\ 3x - y = 5 \end{cases}$

b) $\begin{cases} \frac{9}{5}x + \frac{6}{5}y = 9 \\ 9x + 6y = 39 \end{cases}$

50. An airplane flying into a headwind travels the 1800-mile flying distance between two cities in 3 hours and 45 minutes. On the return flight, the airplane travels this distance in 3 hours. Find the airspeed of the plane and the speed of the wind, assuming that both remain constant.

51. Simplify the expressions:

a) $\frac{(7x+1)^3(48x^2+5)-(8x^3+5x)(6)(7x+1)^2(7)}{[(7x+1)^3]^2}$

b) $\frac{-2(x^2-8)^{-3}(2x)(x+2)^3-3(x+2)^2(x^2-8)^{-2}}{[(x+2)^3]^2}$

c) $\frac{(x+7)^{3/4}(x+5)^{-2/3}-(x+5)^{1/3}(x+7)^{-1/4}}{\left[(x+7)^{3/4}\right]^2}$

52. Rewrite the quadratic portion of the algebraic expression as the sum or difference of two squares by completing the square:

a) $\frac{4}{x^2+6x+73}$

b) $\frac{1}{\sqrt{63+2x-x^2}}$